MUSURGIA RECORDS

THE THEORY OF CLASSICAL GREEK MUSIC

By FRITZ A. KUTTNER with the assistance of J. MURRAY BARBOUR

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INTRODUCTORY NOTES

I. DEFINITION OF PITCHES AND INTERVALS IN CENTS.

In musical terms an interval is the distance between two tones, or the difference between two pitches, regardless of whether the two tones or pitches are sounded simultaneously (chord) or in sequence (melodic step). In the rather rigid system of modern Western intonations intervals are usually identified by names referring to standardized steps in equal temperament, e. g.: whole tone, semitone, major/minor third, perfect fifth, diminished fifth, etc. Occasionally quartertones may be mentioned, the only deviation from the chromatic series of twelve semitones in equal temperament which form the modern Western tone system.

In acoustical and mathematical terms intervals are defined by the ratio of two figures representative of the two pitches. These figures can be divisions of strings (on a measuring device of ancient origin called monochord), lengths of sonant tubes in wind instruments, or the number of acoustical vibrations (cycles) per second (cps) which produce a given pitch, for example:

Ratio of two string lengths 12 : 18 inches (=2:3, the ratio of the perfect fifth)

Ratio of two vibration numbers 440 : 880 cycles per second (=1:2, the ratio of the perfect octave)

Unless the numerical ratios are very simple ones—such as above—it is very hard to visualize, without calculations, the size of the intervals thus expressed. The ratios 17:18 (approximately a semitone in equal temperament) or 587:740 cycles (a major third in equal temperament) do not convey any immediate idea of the intervals concerned. Absolutely confusing are ratios such as 531,441 : 524,288. Yet, this ratio stands for the Pythagorean comma, one of the most important theoretical and practical problems in Western music for almost two thousand years.

In 1885, Alexander J. Ellis, an English scholar, proposed an improved system of calculation which gives an immediate and clear description of the interval. It is of particular value for the definition of micro-intervals and of intervals deviating from the standard of Western pitches in equal temperament. Today Ellis' system is generally accepted and an indispensable tool for all inquiries into musical acoustics, historical intonations, "exotic", primitive and ancient musical scales.

In Ellis' method of logarithmic calculation the interval of an octave is equal to 1,200 cents, each of the twelve (tempered) semitones measures 100 cents. Thus, the interval c—c♯ equals 100 cents, c—d=200 cents, the major third= 400 cents, the fifth c—g, or any fifth for that matter, =700 cents, etc.

If we are told that the Pythagorean comma measures 24 cents, (cf. Table no. 1, col. 7), we can visualize this small interval immediately as a pitch difference of approximately one quarter of a (tempered) semitone. The information that a perfect acoustical fifth equals 702 cents, makes it immediately clear that in equal temperament the perfect fifth is lowered by the micro-interval of two cents, bringing it down to 700 cents. An interval measuring 911 cents, thus, is easily visualized as being 11 cents sharp as compared with the major sixth in equal temperament.

The above mentioned difference of 24 cents, the Pythagorean or Ditonic comma, is an "impurity" and shortcoming of the "Pythagorean" tone system which the theorists of many centuries tried to eliminate or to overcome by all kinds of adjustments and compromises which were called temperaments. The system generally accepted in modern Western music is that of **equal** temperament which sacrifices the perfectly intoned fifth of 702 cents and substitutes twelve equal and slightly flat fifths at 700 cents, resulting in twelve equal semitones of 100 cents each.

Equal temperament has many disadvantages, but it made possible the enormous sophistications of modern harmony in Western music, and the easy modulation from one key into another.

Anyone able to handle simple logarithmic calculations and logarithmic tables can learn to calculate intervals in cents. The article on "Intervals, calculation of", in Willi Apel's Harvard Dictionary of Music, contains a simplified method and formula for the conversion of interval ratios into cents*). A more detailed discussion of the topic and various methods of calculation are given in the excellent appendices which Alexander J. Ellis wrote for his translation of Helmholtz' work on "The Sensations of Tone". (A recent reprint of the work was issued by Dover Publications, Inc., New York, 1955. Here Ellis' discussion will be found in Appendix no. xx, Section C, pp. 446 ff.)

The reader is encouraged and urged to try his hand on such calculations; it is much easier than it sounds. One or two hours of practice will make Ellis' method a valuable and ready tool for a lifetime.

II. A SHORT SURVEY OF THE HISTORY OF GREEK MUSICAL THEORY.

The "Pythagorean" tone system which formed the fundament and point of departure for most of Greek musical theory, was probably not completely worked out until Euclid, or some disciples of the Pythagorean school shortly before Euclid, closed mathematically the first octave orbit by a succession of acoustically perfect fifths.

The principle of building a tone system within the compass of an octave by a series of successive fifths is a very ancient one. There is sufficient archeological evidence to credit the Sumerian civilization with a pentatonic system derived from **four** upward steps of successive fifths, in the middle of the fourth millennium B.C.

C———G———D———A———E
1 2 3 4

This scale has, as its only intervals, major seconds (c-d, d-e, g-a, and minor thirds (e-g, a-c) at 204 and 294 cents respectively.

Old Babylonia had a similar system around 2500 B.C.; so had the Egyptians during that same period. There is no doubt that most of the early Greek achievements in musical theory and practice were taken over from sources in Western Asia, among them the series-of-fifths system. When and where any West Asian music civilization first went beyond the fourth step shown above, to create a scale containing semitones, is not known. But we can be sure that in Pythagoras' times, around the middle of the sixth century B.C., a tone system

*) Useful tables for conversion of frequencies into cents and vice versa have been compiled by Robert W. Young and published by C. G. Conn, Limited, in Elkhart, Indiana, 1939.

was known within the orbit of Greek civilization that went beyond four cycle steps. It is likely that—by then—six steps had been completed, adding b and f♯ to the above series and producing a scale with two semitonic steps at 90 cents each:

	C	D	E	F♯	G	A	B	C
Pitches:	0	204	408	612	702	906	1110	1200 cents
Intervals:	204	204	204	90	204	204	90	

Then, some time in the fifth century B.C., the cycle was closed for the first time **in Mediterranean culture,*)** producing twelve semitones of 90 and 114 cents alternately within the octave and revealing the nature and size of the "Pythagorean" comma. From then on the various keys and modes of the Greek scales could be developed, along with rules for modulation from one key into another. Much thought has been given by scholars to the question as to whether or not the variety of musical **practice** came first, and the mathematical calculations of the tonal material already in actual use followed only as a theoretical rationalization **post factum.** We are inclined to believe that the two processes went on simultaneously all the time: The theoretical discussions and computations of the mathematicians and philosophers must have influenced, ordered and solidified musical practice which, on its part, kept on supplying theory with new problems and techniques to speculate on.

Pythagoras lived and worked in **Tarentum.** The school and religious sect founded by him had its focal point for more than 200 years in the same colonial center of Greek culture. The Pythagorean Archytas of Tarentum discovered, around 380 B.C., that vibrations of air and other sonorous media were the source of sound and tones, thus paving the way for intervals based on acoustical laws rather than arithmetical computations. Another resident of Tarentum, Aristoxenos, wrote exhaustively on melodic and rhythmic problems around 330 B.C. Obviously continuing where Archytas had left off, he stressed the postulates of the hearing sense as opposed to the numerical theories of the Pythagoreans.

Hereafter the center of gravity in acoustical and musical scholarship shifts from Tarentum to Alexandria, another important deposit of Greek colonial culture. Around 300 B.C. Euclid may have completed his "systema telaion", the "perfect system" of tetrachords, melodic, scale, and modal structure of music in his time. Around 230 B.C. Eratosthenes of Alexandria contributes further to the tetrachord theory and mathematical scale structure. But then it takes almost 200 years until another great theorist completes a step of lasting importance: Didymos of Alexandria (ca. 30 B.C.) None of his writings are preserved; what we know about his theoretical work, stems from the reports about Didymos in the writings of Ptolemy. Many history and text books credit Didymos with the introduction of the major third in just (or natural) intonation into Greek tetrachord theory, basing this information on Ptolemy. This is an error. As will be seen in the following pages, the natural third occurs as early as ca. 380 B.C. in the enharmonic tetrachord of Archytas (cf. example no. 12, column 19). Thus Ptolomy's error is being perpetuated in modern texts.

It would appear, however, that Didymos was the first one to realize the superiority of **small** superparticular ratios (explained in cols. 9-10) over certain Pythagorean intervals. He may also have been the first theorist who heard or "sensed" a number of harmonics and related them to small superparticular ratios—a consequence which had been prepared by the work of his Tarentian predecessors Archytas and Aristoxenos.

Another major achievement traditionally attributed to Didymos is the discovery of a larger (major) and a smaller (minor) whole tone, and—immediately connected with this discovery—the realization of the pitch difference between the Pythagorean and the natural major third. This difference, 22 cents in size, is called the syntonic (or sometimes the Didymic or Ptolemaic) comma. Again, there is much reasonable doubt that this discovery should have been made as late as in the time of Didymos. It is possible, however, that he was the first to find the mathematical expressions for these differences.

*) In Far Eastern civilization the circle had been closed, both in practice and theory, much earlier. Sonorous stones found in the Princes of Han tombs in Lo-Yang, China, have the precise intonation of a complete "Pythagorean" circle. The stones must be dated prior to 550 and possibly as early as 900 B.C.

Another 170 years later the last of the great theorists of Greek colonial music culture makes his contribution. Around 140 A.D. Ptolemy of Alexandria gives in three famous books a comprehensive survey of Greek theory on scales and intervals. He stresses the supremacy of the diatonic-syntonic genus with its natural major third 4:5 (386 cents) and the natural minor third 5:6 (316 cents). This sets the stage for the subsequent developments of medieval music theory in Western Europe.

A few conclusions are possible even from this sketchy review of Greek musical theory:

(1) Prior to 380 B.C. there is no evidence of a successful attempt towards penetrating scientific analysis in Greek Mediterranean culture. Consequently, before that time, Greek music must have been based mainly on practice alone, and could not have been really independent from West Asian practice and theory. Greek philosophy and reasoning prior to 380 B.C. were preoccupied mainly with the educational, ethical, and political functions of music, i. e. with the social aspects of the art rather than its scientific and theoretical fundaments.

(2) All important Greek writers on musical theory lived and worked either in Tarentum or in Alexandria. With only a few exceptions, all important Greek musicians and other artists who contributed in pre-Euclidian times to the **practice** of Greek music, lived and worked either in the coastal towns of Asia Minor or on the colonial islands in the Aegean Sea near the Asia Minor coast (mainly Lesbos and Samos). One is tempted to ask what the role and importance of Greek music and musical theory actually was, in the five centuries before the birth of Christ, on the Greek peninsula itself.

(3) The influence of Egypt and the Near East on "Greek" music theory must have been very strong during the time from 300 to 30 B.C. when the center of theoretical scholarship remained in Alexandria.

(4) Trained on humanistic principles and Hellenistic conceptions of European culture in high school and college, we are likely to assume that Greek music was the fundament of Western or European music, and that it should be similar to it. In the following attempt to reconstruct some of the actual sonorities of Greek music, we are going to experience a surprise: it is the sound of the Orient, of West Asia, that speaks to us in this recording—a sound to be sure that has been theoretically rationalized by an early mixture of Asian and Mediterranean philosophies.

III. METHODS USED FOR THE PREPARATION OF THIS RECORDING

The most important decision in the planning of this disk was the selection of the sonant medium: which instrument available in our time was best suited for the reproduction of tone phenomena of ancient Greek music? Both historical and practical considerations were involved in that decision.

The tone quality selected would have to come reasonably close to that of any of the important instruments in Greek music, such as kithara, lyre, aulos. This consideration excluded the piano, the organ, trombone, etc. Practical requirements were the following:

The instrument's pitches must permit of easy and precise adjustment to any desired micro-intervallic changes. (That excluded all modern woodwinds.) The instrument had to be capable of holding any pitch at great precision for a reasonable length of time—sufficient anyway to make a recording—and it had to allow the playing of scales and short musical pieces within at least the range of two octaves at precisely the pitches needed for each individual tone. Finally the instrument had to produce sufficient tone volume to permit precise tuning, and to make an intelligible recording clear enough for highly concentrated listening.

These requirements narrowed down the variety of given possibilities to a stringed and plucked keyboard instrument of the harpsichord type. The final selection favored a small practice instrument by John Challis with only one string per tone and key. That facilitated the tuning process as compared with multistringed harpsichords. It also eliminated the thick, resonant and reverberating tone of the oversized modern concert harpsichords.

All tunings for the recording were done with the aid of a stroboscopic frequency meter*) which permits accuracy within one cent. Each individual intonation was double-checked before and after recording to eliminate possible changes of pitch during actual recording. Thus, all pitches and intervals are certain to be precise within a deviation tolerance of ± 1 cent. This ratio of accuracy will not be influenced by the precision of the turntable used for the playback as far as interval **ratios** are concerned, (with the exception of permanent strong fluctuations in the speed of a very inferior turntable producing noticeable flutter or "wow").

No one would want to argue that the harpsichord used for this recording permits a true reproduction of the tone qualities of a Greek lyre or kithara. But it comes as close to the tone of these plucked string instruments as all other important requirements permit. Modern lute or guitar type instruments had to be excluded because of their inability to allow high-precision tunings for more than one full octave's range.

An effort has been made to keep a' the center of reference for all intonations at 440 cps for an absolute pitch. Temperature and other influences may change this absolute pitch within ± 4 cents or ± 1 cps. Thus playback should produce the reference tone a' always between 439 and 441 cps, i. e. reasonably close to modern standard piano tuning, if the speed of the turntable stays close enough to 33 1/3 rpm. All intervals and **relative** pitches, however, will remain precise within ± 1 cent, no matter how large the deviations of the **absolute** pitch should get during playback.

All scales recorded follow the practical method selected by J. M. Barbour, i. e. the Dorian octave from e'—e, downward on the white keys of the piano keyboard. The demonstration of all scale examples uses a metric pattern calculated to accentuate the structure of each individual tetrachord, with the disjunct major second interval always between b and a, and always in the ratio 8:9, = 204 cents. All scales are played downward, because the Greek system, in its Euclidian version ("systema telaion", the perfect system), was conceived as a descending scale. All scale examples extend over two octaves, from e" to e, in order to give two symmetrical octaves with two symmetrical disjunct tetrachords each, in every octave:

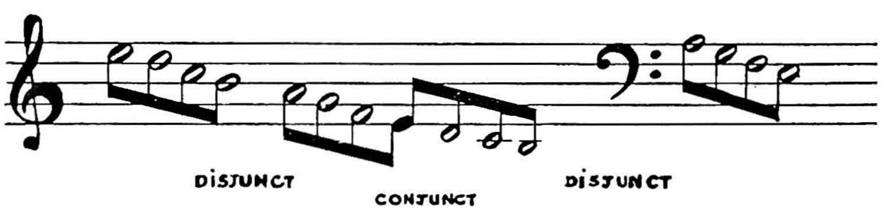


Fig. 1

This presentation deviates from the procedure followed by Curt Sachs and other authors who present the "perfect system" in the compass from a' to A downward in the Dorian octave. The reasons for this departure were purely practical ones, such as permitting the student a comparison with Barbour's listings within the range from e' to e. No differences of theoretical or sound interpretation arise from this departure: it still remains a Dorian octave, only on a different level of reference pitch. As we do not know anything about what a possible standard pitch—if any— might have been at any time in Greek history, the whole question is irrelevant from a theoretical point of view.

IV. LISTENING TO THE RECORDING:

The reader is cautioned that listening to unusual intervals and micro-intervals is a matter of acquired skill and ear training which calls for a certain amount of practice and concentration. Our modern ears have become lazy and indifferent by contemporary listening diet which consists of nothing but equal temperament intonation and its admixtures of "Pythagorean" and natural intonations. These admixtures vary in degree at any moment during performance as we experience

*) The "Stroboconn", courtesy Messrs. C. G. Conn, Ltd., Electronics Division, Elkhart, Indiana.

it in our concert halls. We do not care very much, nor do we notice consciously, whether we hear a natural, a "Pythagorean" or a tempered interval in actual performance.

This does not mean that a major skill has to be developed to hear and identify micro-intervals, or micro-intervallic deviations from true equal temperament intonation. But a few hours of concentrated listening and many repeated hearings of the demonstration examples will normally be required, before a clear and sensitive definition of the sound phenomena begins to form. The results of this short period of training and preparation are always gratifying: a new sense of interval and intonation values develops, together with a keenness of discrimination which is very useful to the singer and instrumentalist alike, and highly stimulating to the critical observer and listener. **The net gain produced by one playing of this record will be negligible; ten or more hearings will begin to open a new world of tone sensations.**

Clicking Noises:
Occasionally, there will be heard slight clicking noises in the examples, especially during slow demonstration of single tones such as in scales and tetrachords. These clicks are caused by the harpsichord's jacks falling back into position after the string is sounded. As greatest clarity of all sound phenomena was of supreme importance, the microphones had to be placed very close to the strings. Thus it could not be avoided that clicks were occasionally picked up by the microphones. The alternative—removing the microphones to greater distances—was impractical because this would have severely impaired the clarity of many sounds. In continuous harpsichord performance at normal speeds the sound of the jacks is completely covered by the tone volume of the music itself.

V. REFERENCES:

J. Murray Barbour. Tuning and Temperament. A Historical Survey.
Michigan State College Press, East Lansing, 1951.
Curt Sachs. The Rise of Music in the Ancient World East and West.
W. W. Norton & Co., New York 1943.
Archibald T. Davison and Willi Apel. Historical Anthology of Music.
Harvard University Press, Cambridge, Mass., 1949.

In preparing the commentaries and the sound aspects of this recording, F. A. Kuttner leaned heavily on Dr. Barbour's text which was one of the most important sources permanently used in the planning and completion of this disk. Dr. Barbour kindly agreed to share responsibility in this undertaking by editing and revising the written commentaries prepared by F. A. Kuttner, and by giving advice and assistance in various stages of the work.

Personalities:

Dr. F. A. Kuttner was Associate Professor for Oriental musicology at the Asia Institute, Graduate School for Asiatic Studies, in New York. He specializes in Oriental, comparative, and archeo-musicology.

Dr. J. Murray Barbour is Professor of musicology at Michigan State University, East Lansing. For many years he has specialized in acoustics and in the history of tunings and temperaments.

Robert Conant is harpsichordist and graduate assistant in instruction at Yale University, New Haven, where he frequently conducts the classes of his teacher Ralph Kirkpatrick during the latter's absence on concert tours.

VI. THE "PYTHAGOREAN" TONE SYSTEM TWELVE CYCLIC STEPS UPWARDS.

The scale produced by this system uses only two intervals: the perfect octave (ratio 1:2, = 1,200 cents), and the perfect or natural fifth (ratio 2:3, = 702 cents). The twelve semitones within the range of an octave are produced by twelve steps of consecutive fifths upwards. Whenever a resulting tone exceeds the compass of one octave, it has to be brought back into this compass by octave transposition, i. e. by subtracting 1,200 cents, or by multiplying by 2, the ratio of the octave. The reader is advised to compute these simple arithmetic examples himself to get a clear impression of the acoustical proportions involved in this system.

TABLE 1

Cyclic Step No.	Resulting Tone	Cyclic Ratio	Converted into cents
0	C	1 (see note below)	0
1	G	2 : 3	702
2		(2:3)×(2:3)	+ 702
			1404
2 (a)		(4:9)× 2	− 1200
	D	8 : 9	204
3		(8:9)×(2:3)	+ 702
	A	16 : 27	906
4		(16:27)×(2:3)	+ 702
		32 : 81	1608
4 (a)		(32:81)× 2	− 1200
	E	64 : 81	408
5		(64:81)×(2:3)	+ 702
	B	128 : 243	1110
6		(128:243)×(2:3)	+ 702
		256 : 729	1812
6 (a)		(256:729)× 2	− 1200
	F♯	512 : 729	612
7		(512:729)×(2:3)	+ 702
		1024 : 2187	1314
7 (a)		(1024:2187)× 2	− 1200
	C♯	2048 : 2187	114
8		(2048:2187)×(2:3)	+ 702
	G♯	4096 : 6561	816
9		(4096:6561)×(2:3)	+ 702
		8192 : 19683	1518
9 (a)		(8192:19683)× 2	− 1200
	D♯	16384 : 19683	318
10		(16384:19683)×(2:3)	+ 702
	A♯	32768 : 59049	1020
11		(32768:59049)×(2:3)	+ 702
		65,536 : 177,147	1722
11 (a)		(65,536:177,147)× 2	− 1200
	E♯	131,072 : 177,147	522
12		(131,072:177,147)×(2:3)	+ 702
	B♯	262,144 : 531,441	1224
12 (a)		(262,144:531,441)× 2	− 1200
	C	524,288 : 531,441	24

This last tone C is actually B♯ transposed one octave down. The ratio for C (see above, first step) from which we started out, was 1, (or 1 : 1) while the ratio resulting from step no. 12(a) is a tiny fraction smaller than 1, hence somewhat sharp (24 cents) as against C. This fraction is the Pythagorean (also called the Ditonic) comma. The higher octave C' would have the ratio 1:2 = .5, i. e. half the string length required for sounding the lower octave C. Thus any ratio smaller than 1 represents an interval higher than C, any ratio larger than 1 represents a tone lower than C.

Note: The starting ratio for the tone from which we set out (1:1 = 1) stands for ONE unit of whatever medium we choose for measuring pitches, for instance a string 1 yard long, or a sonant tube length of 2 feet, or the standard tone a'=440 cycles per second (cps). If, e. g., we equate 440 cps with this unit 1, the higher octave a" would sound at 880 cps. The major second b' above a' would be computed as follows:

ratio for major second, C-D,
as per step no. 2 (a) : 8 : 9

$$\frac{8}{9} = \frac{440}{x}$$

$$x = \frac{9 \times 440}{8} = 495 \text{ cps.}$$

This value b' = 495 cps is, of course, correct only if we wish the tone b' to sound in Pythagorean intonation. In equal temperament tuning the frequency for b' would be 493.88 cps, if the standard tone a' has 440 cps as reference basis.

VII. THE "PYTHAGOREAN" TONE SYSTEM IN MEDIEVAL AND RENAISSANCE USAGE

Above computation consisting of twelve consecutive steps upwards is the original system as erroneously ascribed to Pythagoras. It was probably not completed up to the twelfth cyclic step until about 100 years after Pythagoras (see above, col. 3). Thereafter, however, a complete cycle of twelve steps upwards was traditional practice for the "Pythagorean" system in Greek theory.

In the Middle Ages and the Renaissance period the system was used in a different way: only seven or eight cyclic steps were computed upwards, and three or four cyclic steps were computed downwards from C. Oddly enough, most textbooks show a "Pythagorean" circle with six steps upwards, and six steps downwards where the Pythagorean comma appears as the difference between the sixth upwards step F♯, and the sixth downward step G♭. It should be noted, however, that this arrangement of six-up, six-down was never used in Euroepan practice. (See Fig. 2-4).

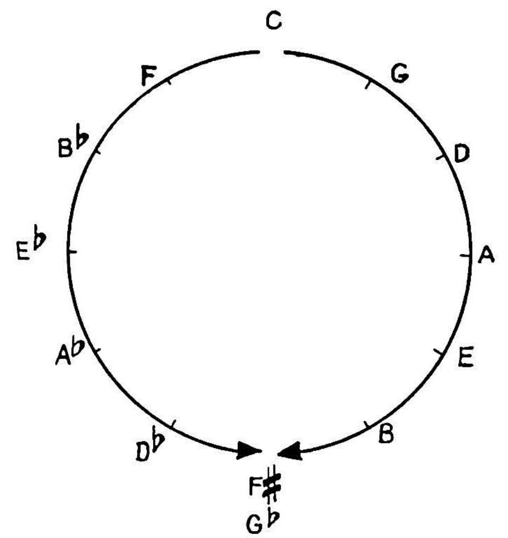
The following table shows the computation for six steps downward:

TABLE 2

Cyclic Step No.	Resulting Tone	Cyclic Ratio	Converted into cents
0	C	1	1200
1			− 702
	F	3 : 2	498
2 (a)		(3:2)×(1:2)	+ 1200
		3 : 4	1698
2		(3:4)×(3:2)	− 702
	B♭	9 : 8	996
3		(9:8)×(3:2)	− 702
	E♭	27 : 16	294
4 (a)		(27:16)×(1:2)	+ 1200
		27 : 32	1494
4		(27:32)×(3:2)	− 702
	A♭	81 : 64	792
5		(81:64)×(3:2)	− 702
	D♭	243 : 128	90
6 (a)		(243:128)×(1:2)	+ 1200
		243 : 256	1290
6		(243:256)×(3:2)	− 702
	G♭	729 : 512	588

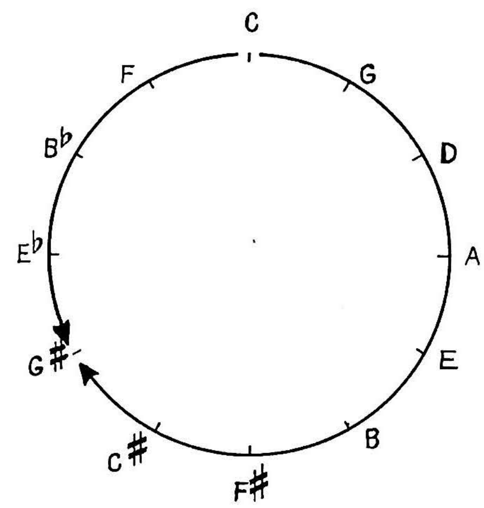
F♯ = 612 cents
G♭ = 588 cents

Comma = 24 cents



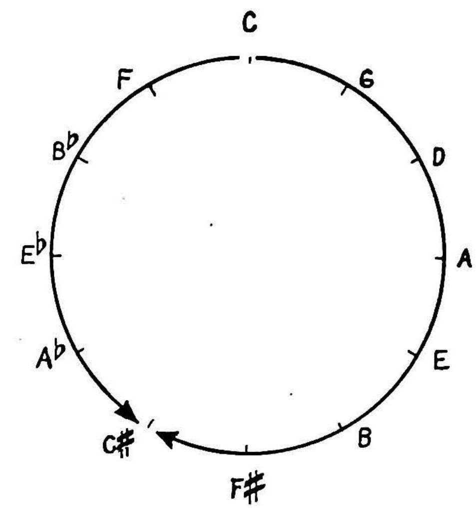
6 cyclic steps up
6 down

Fig. 2



8 cyclic steps up
3 down

Fig. 3



7 cyclic steps up
4 down

Fig. 4

All above ratios are in the octave range **below** our starting tone C. In order to bring the resulting tones into line with the tones gained in the cyclic "upwards series" (Table no. 1), they have to be transposed one octave **up,** by multiplying each of the "downward ratios" by (1:2). Thus the G♭ of the above sixth step would become in the higher octave 729:1024, or converted into cents, = 588 cents. **Note:** All cyclic steps marked (a) in the above two tables indicate that here an octave transposition is taking place. The step numbers without an (a) represent another cyclic step of a perfect fifth.

VIII. THE DIVISIVE SYSTEM

Apart from the cyclic (or "Pythagorean") principle for scale building, there was another method known and used in early Asian and Greek music theory which is usually called the **Divisive Principle.** It is based on the realization that a number of important intervals can be constructed by simple numerical ratios of neighboring figures; e. g. : 1:2 = octave; 2:3 = perfect fifth; 3:4 = perfect fourth; 4:5 = major third; 5:6 = minor third; 8:9 = major second, etc. Some of these ratios are identical with the Pythagorean cyclic ratios, viz. 1:2; 2:3; 3:4; 8:9.

Such fractions of neighboring figures are called **superparticular ratios.** They have played an important part in ancient music theory, frequently for reasons of numerical mysticism or superstition. In modern times they continue to influence theoretical thought because these ratios are the mathematical expressions of the series of harmonics sounding in most musical tones besides the frequencies of the fundamental tone. If, e. g., a tone is sounded on any instrument at a frequency of 100 cps, its second, third, fourth etc. harmonics have the frequencies of 200, 300, 400 etc. cps, and farther up to as much as 1600 or 2000 cycles; sensitive measuring equipment permits to prove occasionally the presence of 16 to 20 harmonics with certain fundamental tones.

The discovery, in modern times, of numerous harmonics and of the fact that their respective frequencies are represented by superparticular ratios, led quite naturally to renewed speculation on the merits of intervals constructed on the basis of such ratios. Especially physicists and mathematicians showed a tendency to overestimate the value of such intervals for practical musical purposes. Under normal circumstances the human ear cannot hear and distinguish more than four or five harmonics with any given fundamental, and the audible or perceptible maximum appears to be seven or eight partial tones as a rare exception. Consequently, superparticular ratios beyond the limit 7:8 cannot have **practical** musical value. As will be seen and demonstrated in the recording, there is even reason to doubt the musical merit of a superparticular ratio as low as 4:5. Ratios such as 27:28, or 45:46 have no plausible bearing on practical and useful scale structures. They are hardly more than theoretical dogmatism.

Note: There is some confusion about the order in which interval ratios should be spelled out. Some authors use the order 2:1; 5:4; 81:64. Others prefer it the other way around: 1:2; 4:5; 64:81. There is no real difference between the two spellings; the choice depends on whether we think **first** of the higher or the lower tone of any interval, or whether reference is made to the ratios of string lengths (2:1) or acoustical frequencies (1:2). This writer prefers to think in terms of acoustical frequencies and therefore sets the lower tone first. It should be noted, however, that **for logarithmic conversion of ratios into cents** the higher figure has always to come before the smaller one. In logarithmic procedure, division becomes subtraction; thus the smaller logarithm has to be subtracted from the larger one to avoid negative logarithms which have no meaning in the calculation of intervals in cents.

COMMENTARIES TO THE RECORDED EXAMPLES

First Group: "PYTHAGOREAN" INTONATION IN THE PRACTICE OF THE MIDDLE AGES AND RENAISSANCE PERIOD: EIGHT CYCLIC STEPS UPWARDS, ENDING WITH G♯, AND THREE STEPS DOWNWARD, ENDING WITH E♭.

If we line up, in the order of a chromatic scale, the various tones calculated in the two Pythagorean tables, taking the first eight intervals from table no. 1, and the first three intervals from table no. 2, we get a "Pythagorean" chromatic scale as it was used during the Medieval and Renaissance periods in Central and Western Europe.

TABLE 3

Tone	"Pythagorean" Intonation	Interval Difference	Modern Equal Temperament For Comparison		
C	0		0		0
		114			
C♯	114		100	plus	14
		90			
D	204		200	plus	4
		90			
E♭	294		300	minus	6
		114			
E	408		400	plus	8
		90			
F	498		500	minus	2
		114			
F♯	612		600	plus	12
		90			
G	702		700	plus	2
		114			
G♯	816		800	plus	16
		90			
A	906		900	plus	6
		90			
B♭	996		1000	minus	4
		114			
B	1110		1100	plus	10
		90			
C'	1200		1200		0

An evaluation of this scale shows that there are two different sizes of semitones, at 90 and 114 cents alternately, which make this scale unfit for a wide range of modulations, and ambiguous for chromatic progressions.

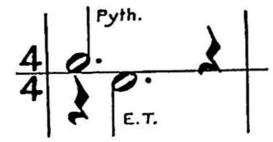
In the following seven examples the intonation used throughout is "Pythagorean", with eight steps upwards (to G♯) and three steps downwards (to E♭), as shown in Table no. 3. (See also Fig. 3).

The recording begins with the demonstration of three diatonic scales in this tuning. In each of these scales the seven degrees are first played by themselves (a), then repeated by comparing each tone in "Pythagorean" pitch with its equivalent in equal temperament (b). Finally each degree is sounded together with the tonic, first in "Pythagorean" intonation, then for comparison in equal temperament (c).

EXAMPLE No. 1.
Diatonic Scale in C-Major.
Section a.

Tones:	C		D		E		F		G		A		B		C'
Cents:	0		204		408		498		702		906		1110		1200
Intervals:		204		204		90		204		204		204		90	

Section b.
The first tone of each pair of tones is in Pythagorean intonation, the second tone in equal temperament. All pairs are played in the following rhythmical pattern



which permits the development of beats during the overlapping middle part, counts 2 and 3, and provides for separate listening on counts 1 and 4. This same pattern is applied on all further occasions whenever micro-intervals are demonstrated for comparison.

C : 0 - 0. D : 204 - 200. E : 408 - 400. F : 498 - 500.
G : 702 - 700. A : 906 - 900. B : 1110 - 1100. C' : 1200.

Section c.

C - D	:	0 - 204;	0 - 200.	C - E	:	0 - 408;	0 - 400.
C - F	:	0 - 498;	0 - 500.	C - G	:	0 - 702;	0 - 700.
C - A	:	0 - 906;	0 - 900.	C - B	:	0 - 1110;	0 - 1100.
C - C'	:	0 - 1200.					

In this medium range of the keyboard, interval differences of two or three cents are usually inaudible to even the finest ears. It is assumed that such small micro-intervals could not even be "sensed" by the beat that develops when closely neighbored frequencies are sounded simultaneously. According to authorities on tone physiology, keen and trained ears usually begin to distinguish micro-intervals in this range from four cents on upwards.

On the tones F and G (two cents each) supposedly no one should be able to distinguish the micro-tonic difference. It appears, however, that a number of well-trained ears do hear the difference on the perfect and tempered fifths 702—700 cents. This writer, for example, believes that he can hear it—as the only case of micro-intervallic discrimination below three or four cents in this range. If this be correct, we may conclude that our lifelong training in hearing and tuning perfect fifths on stringed instruments helps to develop this extraordinary acuity, and for this exceptional case of the perfect fifth only. We may also conclude that acuteness of pitch discrimination is more subject to training than to "natural physiological limitations", as is widely believed.

Beginners who do not hear, at their first attempts, the four cents interval on the tone D, or even the six cents distance on the tone A, should not get discouraged. It takes a certain amount of practice to develop this type of pitch discrimination. The capacity of distinguishing micro-intervals between three and four cents is rarely found among persons of the Western races; the nations of East Asia appear to have a finer hearing sense for this kind of differences because of their languages where tiny tone inflections add different meanings to many words or syllables of the same pronounciation.

The distance of eight cents on E, and the ten cents difference on B, however, should be evident to any average ear; if it is not perceived at the first attempt, a few repeated and concentrated listenings will produce a distinct perception.

The "Pythagorean" scales in G-Major, D-Major, and A-Major are composed of precisely the same intervals as the key of C-Major above:

204 — 204 — 90 — 204 — 204 — 204 — 90 cents.

(Compute these three keys yourself from Table no. 3 to confirm this fact!)
Thus, a sounding demonstration of these three keys could not add anything new to the previous example.

EXAMPLE NO. 2.
Diatonic Scale in E-Major.
Section a.

Tone:	E	F♯	G♯	A	B	C♯'	E♭'	E'
Cents:	408	612	816	906	1110	114	294	408
Intervals:	204	204	90	204	204	180	114	

This scale introduces two new intervals, a (minor) whole tone of 180 cents, and a very wide semitone of 114 cents. Here the Pythagorean (or Ditonic) comma takes effect for the first time at the point of the enharmonic change between E-flat and D♯. For the key of E-Major the seventh degree calls for the D♯ tuning = 318 cents (cf. Table no. 1, step no. 9a), but in the intonation selected by us we have, instead, the third downward step resulting in E♭ = 294 cents (cf. Table no. 2, step no.3). The difference is a full comma of 24 cents which throws the last two degrees of the scale off balance: 180 cents instead of 204, and 114 cents instead of 90.

The example shows clearly that beyond the limit of three sharps Pythagorean tuning becomes a questionable method for Western music—even in simple monophonic settings or in primitive early polyphony.

Section b.
E : 408 - 400. F♯ : 612 - 600. G♯ : 816 - 800. A : 906 - 900.
B : 1110 - 1100. C♯' : 114 - 100. E♭' : 294 - 300. E' : 408 - 400.

This example shows very strong deviations from equally tempered intonations. Six cents is the smallest difference occurring (on A and E♭) while all other degrees show differences of 8, 10, 12, 14 and 16 cents. They are easily heard even by untrained ears, and gradually a perception of quantity will develop which will distinguish an eight or ten cents difference from a 14 or 16 cents distance.

Section c.
E :408 - 400. E - F♯ : 408 - 612; 400 - 600.
E - G♯ : 408 - 816; 400 - 800. E - A :408 - 906; 400 - 900.
E - B : 408 - 1110; 400 - 1100. E - C♯' : 408 - 114; 400 - 100.
E - E♭' : 408 - 294; 400 - 300. E - E' : 408 - 408; 400 - 400.

The question may be asked whether a semitone of 90 cents or of 114 cents is preferable to our modern ears. A limited number of listening tests made by this writer would seem to indicate that preference is given to the narrow interval, especially on the seventh degree. Modern intonation practices appear to favor a high leading tone, conceivably still under the influence of Pythagorean tuning: 1110 - 1200 = 90 cents. (Cf. Table no. 3.) It has also been argued that a narrow semitone is more clearly defined and more clearly distinct from the various whole tone intervals (cf. Examples no. 32, 33, 35.) than a step of 114 cents. It is hoped that future tests, numerous enough to provide a sound statistical basis, will contribute more definite answers to this and similar questions.

EXAMPLE NO. 3.
Diatonic Scale in B-Major.
Section a.

Tones:	B	C♯'	E♭'	E'	F♯'	G♯'	B♭'	B'
Cents:	1110	114	294	408	612	816	996	1110
Intervals:	204	180	114	204	204	180	114	

In this key we get even two semitones of 114 cents each, and two (minor) whole tones of 180 cents, while the "Pythagorean" semitone of 90 cents disappears altogether. Two commas haven taken effect, the first at the same place as in the previous example, the second at the tone B♭. The latter is an enharmonic substitution for the seventh degree A♯ which is actually needed for the key of B-Major and should properly sound at 1020 cents (cf. Table no. 1, step no. 10). Such is the price paid for the necessarily inflexible tuning of a keyboard instrument or a fretted string instrument. Once it was decided to tune eight steps upwards and three steps downwards to complete a cyclic tuning, there was no other alternative left. If we wanted an improved intonation, the whole instrument would have to be re-tuned, but this would not help very much either: the point of the harshest dissonant clashes would just be shifted to some other degree of the scale.

With its two different whole tones (204 and 180) and its very wide semitones (114 instead of 90), this key has an outspoken "Oriental" character for our modern Western ears, as will be seen later on.

Section b.
B : 1110 - 1100. C♯' : 114 - 100. E♭' : 294 - 300. E' : 408 - 400.
F♯' : 612 - 600. G♯' : 816 - 800. B♭' : 996 - 1000. B' : 1110 - 1100.

From the viewpoint of, and compared with, equal temperament the B-Major tuning could hardly be called inferior to the E-Major intonation in Example no. 2. The total deviation from equal temperament (balancing plus-differences against minus-differences) is:
for B-Major: 60 cents; for E-Major: 68 cents.
(Compute the deviations yourself to confirm these totals!)

This seems surprising at first; the reason for this result is that the two minus-deviations (the enharmonic changes on the third and seventh degrees in B-Major) have a leveling effect on the total deviation from equal temperament. For use in triadic harmony instead of monophonic music, both keys would be very poorly suited.

It should be noted that the Middle Ages never made use of keys as far removed from the center of C, as is B-Major (5 sharps), or of an equivalent number of flats for that matter. In monophony, or in simple two-part polyphony, which dominated most of medieval composition, there was no need for far-removed keys. If, for example, B was desired as a tonic instead of C, all that was necessary from the viewpoint of intonation was to shift the reference pitch of the tonic down for about 15 cycles, and then treat the new pitch and key as if it were basic C-Major.

Section c.
B : 1110 - 1100. B - C♯' : 1110 - 114; 1100 - 100.
B - E♭' : 1110 - 294; 1100 - 300. B - E' : 1110 - 408; 1100 - 400.
B - F♯' : 1110 - 612; 1100 - 600. B - G♯' : 1110 - 816; 1100 - 800.
B - B♭' : 1110 - 996; 1100 - 1000. B - B' : 1110 - 1110; 1100 - 1100.

The next four examples demonstrate the application of the same Pythagorean tuning to a number of Medieval and early Renaissance compositions. As no more accidentals than one flat and two sharps will occur in any of the pieces, all whole tones will measure 204 cents, and all semitones 90 cents.

The main point in presenting the introductory examples no. 1 - 3 was to offer a period of ear training for micro-intervallic pitch discrimination. Pythagorean scales in Medieval or Renaissance intonation seemed as good as any other choice for this purpose.

It should be noted, however, that the (b) and (c) sections in Examples no. 2 and 3 do **not** demonstrate the true pitch deviation of Pythagorean E-Major and B-Major from equal temperament. In order to do so, one would have to use the same reference pitch as starting point for both intonations, i. e. 408 cents for E, and 1110 cents for B. As the sections (b) and (c) in Examples no. 2 and 3 use 400 and 1100 cents as starting point for equal temperament, overlarge deviations between the two tunings are resulting at certain points.

The true sound of the scales, then, is demonstrated in all sections (a), while the (b) and (c) parts will be found useful for listening practice.

EXAMPLE NO. 4.
Parallel Organum. Rex Coeli. (Musica enchiriadis, ca. 850 A.D.) After Davison-Apel, page 22, no. 25.b.2.
The two sections of the piece are individually repeated.

For such simple polyphony our intonation is fully adequate. All fourths and fifths are perfect, at 498 and 702 cents respectively. The major third, in the wide Pythagorean intonation at 408 cents, is not really offensive to our contemporary ears, for reasons to be discussed later.

EXAMPLE NO. 5.
Free Organum. Cunctipotens genitor. (11th century)
After Davison-Apel, page 22, no. 26.a.

The many acoustically perfect octaves, fifths and fourths sound very well in this type of composition—in fact, Pythagorean intonation makes it sound better than equal temperament would do. There is, however, one single major third; it occurs in the last section, eleven notes from the end. To sensitive ears the pitch sounds unpleasant; in this context, with all the other perfect intervals present, the wide Pythagorean third at 408 cents clashes noticeably with the tuning of the fourths, fifths, and octaves. The clash is felt by way of contrast rather than by the characteristics of the interval itself.

EXAMPLE NO. 6
Motet: O Beate Basili, by Jacob Obrecht (1430-1505). Four-part polyphonic setting. After Davison-Apel, page 80, no. 76. Measures 1 - 18.

Three accidentals occur passim in this section of the motet: f♯, c♯, b♭. Thereby modulation takes place between the spheres of G-Minor, D-Minor, C-Major, B♭-Major.

The moment triadic harmony enters abundantly into Renaissance polyphony, the Pythagorean system becomes inadequate and breaks down. While a number of harmonic combinations still sound satisfactory or at least inoffensive, every once in a while certain triads are plainly unpleasant. Oddly enough, these moments occur regularly on a triad of the first degree, e.g. the C-Major chord in measure 5, the B♭-Major chord in measure 8, the double major third in measure 9. Another occasion is the g-g-b♭ in the tenth measure. Again, the final C-Major chord in measure 18 is very unsatisfactory.

In all these combinations, we have the major third at 408 cents and the minor third at 294 cents, adding up to the **perfect fifth** of 702. It is, therefore, clear that **irregularities** of triadic intonation play no part in our displeasure with this kind of triadic harmony. The only explanation, then, is that the major third is too wide, the minor third too narrow **within the perfect fifth**—in other words: the difference between the two thirds is too large for our hearing habits. The minor third heard by itself is found passable; it is the combination of the two Pythagorean thirds into a triad which makes this intonation offensive to modern Western ears.

Many people have wondered why the Middle Ages refused, for such a long period, to recognize the major third as a consonant interval; here is one of several different explanations, and it makes sense: in Pythagorean tuning the major third can, under certain conditions, become a highly irritating tone combination.

Modifications of Pythagorean tuning may have started as early as around 1450; the experiments all aimed for an adjustment of fretted string instruments and keyboard instruments to the new requirements of triadic harmony. It is thinkable that the first experiments of unequal tempering for Pythagorean scales may go back as far as 1400.

It is important, however, to note that for unaccompanied vocal settings, such as the above motet, the problem of a particular temperament or modification of intonation did not arise as early and as urgently as for keyboard music. The vocalist singing in choral groups had always the possibility of adjusting his intonation at any given moment to the circumstances and aesthetic requirements of the score. The advancement of triadic harmony for keyboard and fretted string instruments set the pace for temperament experimentation and gradually, as instrumental accompaniment became common for vocal groups, the vocalists learned to adjust themselves to the limitations of intonation which handicapped the keyboard instruments and subjected them to the necessity of tempered intonations.

EXAMPLE NO. 7.
A dew, a dew. English three-part song, ca. 1500. By Robert Cornysh, (1465-1523). After Davison-Apel, page 90, no. 86.a.

In this simpler triadic setting without polyphonic complexities, the inadequacies of Pythagorean tuning become again evident, and again in particular in simple triads of the first degree. The offensive intonation of thirds stands out most clearly in measures 5 and 6, at the interchanges between C-Major and F-Major, i.e. in the spots where we should least suspect it. What we found on previous occasions per chance, appears to emerge as a principle: to our modern Western ears the Pythagorean major third is less objectionable because of its inherent properties than because it contrasts under certain circumstances with its other tonal surroundings; occasionally it disturbs a delicate balance of intonations, stands out too much and becomes offensive.

There is no certainty, however, that the ears of the musicians in the late Middle Ages or early Renaissance period reacted in the same way as do our own ears in the twentieth century. It will be shown in the following pages and examples that psychological reactions to certain sound phenomena are strongly influenced by training and habit.

* * * * * *

Second Group: GREEK THEORETICAL TUNINGS

CHROMATIC TUNINGS
All Greek scales are constructed, within the compass of one octave, in two equal and symmetrical units of four tones each, the so-called tetrachords. (see above, col. 5). In fully developed Greek tradition there were three different Genera of scales and tetrachords:

The Diatonic Genus; the Chromatic Genus;
the Enharmonic Genus.

The "diatonic" tetrachord and scale is organized as follows:

DESCENDING:	Tone		Tone		Semitone	
Tetrachord I	e''	—— d''	——	c''	——	b'
Tetrachord II	a'	—— g'	——	f'	——	e'

These two symmetrical tetrachords are joined in "disjunction" by the interval of a whole tone (ratio 8:9 = 204 cents) between them, here the interval b' - a'. If we extend this range by one further octave downward, the next tetrachord is joined in "conjunction", the "conjunct" tone e' being common to the two tetrachords no. II and III.

Tetrachord II	a'	g'	f'	e'
Tetrachord III	e'	d'	c'	b
Tetrachord IV	a	g	f	e
	TONE	TONE	SEMITONE	

A Greek **"chromatic"** tetrachord and scale is derived from the "diatonic" genus by moving the second tone of each tetrachord downward by one semitone. Thus, d'' in the above example becomes c♯'', g' becomes f♯', resulting in the "chromatic" tetrachords and scale:

Descending	Minor Third	Semitone	Semitone	
Tetrachord I	c''	c♯''	c''	b'
Tetrachord II	a'	f♯'	f'	e'

EXAMPLE NO. 8
The Chromatic of Archytas (ca. 380 BC.)
Intervals: (Barbour, page 17, table 4)

Tones:	e''——c♯''——c''——b'——a'——f♯'——f'——e'						
Cents:	294	141	63	204	294	141	63
Ratios:			27:28	8:9			27:28

Archytas used here a conventional disjunct interval, the major second b' - a' = 204 cents which we are going to find in **all** tetrachords for the **disjunct** interval; this is part of an inviolable tradition. The "Pythagorean" minor third of 294 cents is also conventional (see Table no. 2, col. 8, for the tone E♭.)

Further analysis shows two strongly different "semitones" at 141 and 63 cents respectively; Archytas introduces here one of the Greek intonation "shades" (Chroai) of the type called "hemiolon" (i.e. one and one half). This creates intervals close to one third and two thirds of a whole tone. Another interval type comes to mind at this occasion:

Throughout the ages and nations the division of the octave into 17 **equal** parts keeps recurring in the history of scale theory and scale experimentation. The result of this 17-division is an interval of 70.6 cents (1200 : 17). The above hemiolic semitones come very close to one and two units of the 17-tone scale, and it may be possible that Archytas could have thought of such a division when he designed this tetrachord.

The effect of Archytas' scale is a rather "weird" or "Asiatic" sound to our Western ears. An attempt at historical interpretation would suggest that in Archytas' time the concept of the "Chromatic Genus" was still a rather flexible one, and that a later generation may have hesitated to call the above scale "chromatic".

EXAMPLE NO. 9

The Chromatic of Didymos. (ca. 30 B.C.)
Intervals: (Barbour, page 18, table 9)

Tones:	e"——	c♯"——	c"——	b'——	a'——	f♯'——	f'——	e'
Cents:	316	70	112	204	316	70	112	
Ratios:	5:6	24:25	15:16	8:9	5:6	24:25	15:16	

Didymos uses a wide minor third of 316 cents as obtained by the superparticular ratio 5:6, or by the augmented second D♯ in Pythagorean tuning (318 cents, see Table no. 1, step 9.a.) rather than E♭ = 294 cents (cf. Table no. 2, step no. 3). His semitones are still very different in size, 70 and 112 cents respectively, but not as close to the chroma hemiolon in Archytas' tetrachord.

EXAMPLE NO. 10.

The Chromatic Syntonon of Ptolemy (ca. 140 A.D.)
Intervals: (Barbour, page 18, table 11)

Tones:	e"——	c♯"——	c"——	b'——	a'——	f♯'——	f'——	e'
Cents:	267	150	81	204	267	150	81	

Ptolemy designs an extremely narrow "minor third" and uses a precise hemiola of 150 cents instead of a semitone. His actual semitone is then reduced to the narrower size of 81 cents. His objective, evidently, was to create a significant interval of 231 cents (150 plus 81) between f♯ and e, or between c♯ and b; this interval is important because it represents a natural major second of the ratio 7:8 = 231 cents.

Furthermore, Ptolemy was strongly preoccupied with superparticular ratios, which he considered as superior to all other interval ratios, especially if they were small. All of the above intervals represent comparatively small superparticular ratios:

267 cents = 6:7; 231 cents = 7:8; 204 cents = 8:9;
150 cents = 11:12; 81 cents = 21:22.

Ptolemy, in the second century A.D., was of course not the first theorist to apply superparticular ratios, but he valued them much higher than the scholars before him, and he had greater mathematical skill to calculate tetrachords with small superparticular ratios than anyone before him.

For comparison, it might be mentioned that all scales of Archytas already contained one such ratio: 63 cents = 27:28. Also, the Chromatic of Didymos is completely made up of superparticular ratios:

5:6 (316 cents); 24:25 (70 cents); 15:16 (112 cents);
8:9 (204 cents).

Didymos, thus, is decidedly a forerunner and competitor of Ptolemy in the question of superparticular intervals.

Historically, the preoccupation with, or the preference for, superparticular ratios, or natural intervals, grew steadily since roughly 200 B.C., and the skill in designing scales with very small ratios developed continuously from that period on. It seems obvious that the acoustical and musical value of natural intervals based on very small ratios had been fully recognized by then; one might even assume that Didymos and Ptolemy, or their contemporaries, knew much more about the harmonic series than we are willing to credit them with. This does not mean, however, that Didymos and Ptolemy acted on purely physical or scientific grounds. As we have seen earlier, ratios beyond the seventh or eighth harmonic have no

more musical value because they can neither be heard nor perceived. Thus, the two Greek scholars must have been influenced by irrational ideas as well, which made them believe in the superiority of any superparticular interval, even if it was too remote from the hearing sense to be of practical significance for musical performance.

From the viewpoint of speculative theoretical dogmatism Ptolemy's Chromatic Syntonon represents a high degree of perfection. What its application and importance may have been for the musical **practice** of Ptolemy's time is hard to guess. We are inclined to have doubts in this respect. The "philosophy" of a tone system played a great part in ancient musical history, both in the Mediterranean area and on the Asian continent. But these philosophies tend to forget about the demands of the musician and the living performance of music which are not easily bound by any kind of formalism.

The only really "orthodox" Chromatic scale which has come down to us from theoretical treatises, is the Chromatic Tonikon of Aristoxenos (ca. 330 B.C.) In his scale the original idea of a chromatic tetrachord

Minor Third plus Semitone plus Semitone

equaling a perfect fourth is really maintained. The intervals in his construction are: 316 plus 93 plus 89 = 498 cents.

As this is not too far removed from modern Western intonation, a sounding demonstration of this scale has not much to offer at this juncture of our studies. The Malakon tetrachord, however, is of considerable interest.

EXAMPLE NO. 11.

The Chromatic Malakon of Aristoxenos (ca. 330 B.C.)
Intervals: (Barbour, page 17, table 5)

Tones:	e"——	c♯"——	c"——	b'——	a'——	f♯'——	f'——	e'
Cents:	379	60	59	204	379	60	59	

For the sake of completeness we list here a number of other "chromatic" tetrachords not demonstrated in the recording:

Aristoxenos' Chromatic Hemiolon (ca. 330 B.C.)	363 plus 69 plus 66 = 498
Eratosthenes' Chromatic (ca. 233 B.C.)	identical with the above Chromatic Hemiolon
Ptolemy's Chromatic Malakon (ca. 140 A.D.)	316 plus 119 plus 63 = 498

The large interval in the intonations of Aristoxenos and Eratosthenes is so close to a **major** third that one can hardly speak of a "chromatic" tetrachord which, in theory at least, requires a **minor** third. Two conclusions might be ventured from this fact:

(I) the idea of the "chromatic" genus must have been highly flexible for quite some time in Greek theory until it was formally frozen in the second or first century B.C.

(II) the "chromatic" genus must have been comparatively unimportant in its **original theoretical** concept and little appealing to the musician and public in this rigid form; the tendency to modify it by all kinds of shades or chroai, and to enlarge the wide interval up to a size of a major third, is too strong to be overlooked.

The two Aristoxenos tunings and the Eratosthenes scale should actually be called enharmonic rather than "chromatic", as will be seen in the following section. The interval of a major third is much more the criterion of the **enharmonic genus** than that of shades such as Malakon or Hemiolon **within** the "chromatic" intonation. Terminology must have varied and changed considerably from the fourth to the third and second centuries B.C., and the frequently expressed idea of a uniform or consequent Greek music theory is thus open to many doubts.

It should be mentioned at this point that all scales and tetrachords of Aristoxenos are of a problematic character for the following reason:

Aristoxenos is the only one among the Greek theorists who computed his tetrachords in "parts" rather than in string lengths. What he meant by "parts", is not at all clear; the meaning has caused heated arguments and inspired wide speculation for many centuries. Ptolemy reports in his writings on the scales of Aristoxenos and gives string length ratios besides the parts for every interval.

But these ratios are Ptolemy's personal interpretation, and we cannot be sure that this interpretation is correct. In the sixteenth century a number of theorists even held that Aristoxenos was the inventor of equal temperament, a fact anyone could try to prove from the parts given for the Diatonic Syntonon of Aristoxenos (cf. Barbour, page 19, table 14).

Tones:	E'		D'		C'		B		A		G		F		E
Parts:		8		8		4		10		12		12		6	

We have here one semitone (4 parts) and two whole tones (8 parts) in the upper tetrachord; in the lower tetrachord the ratio between semitone and whole tone is 6 : 12 parts. Both proportions could suggest that in each tetrachord the whole tones are of equal size and twice as large as the semitone. The ratio between corresponding intervals in both tetrachords is always 2 : 3 (8 : 12, and 4 : 6). Both tetrachords are a fifth apart so that the ratio 2 : 3 could be applied to the proportions between the tetrachords. This looks very much like equal temperament, if we care to choose this kind of interpretation for the meaning of the "parts". In the absence of any proof in support of this assumption we prefer to accept Ptolemy's string length ratios.

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Third Group: ENHARMONIC TETRACHORDS.

The 'enharmonic' tetrachord and scale is derived from the "diatonic" genus by moving the second tone of each tetrachord down a whole tone. Thus, in the original diatonic tetrachord

e"———d"———c"———b'
Tone———Tone———Semitone

d" becomes c", forming a major third with the first tone. The remaining semitonic interval c" - b' is then split up into two quartertones which—as theory usually presents it—are supposed to be equal in size:

Major Third——Quartertone———Quartertone
e"———c"———b'+¼———b'

The "orthodox" enharmonic tetrachord would thus have the following intervals:

(a) with Pythagorean major third: 408 plus 45 plus 45 = 498 cents,
(b) with natural (or just) major third: 386 plus 56 plus 56 = 498 cents;
(c) with equally tempered major third (for comparison only!) 400 plus 50 plus 50 = 500 cents.

FOUR ENHARMONIC INTONATIONS

EXAMPLE NO. 12.

The Enharmonic of Archytas. (ca. 380 B.C.)
Intervals: (Barbour, page 16, table 1)
Tones: e"——c"——c"-¼——b'——a'——f'——f'-¼——e'
Cents: 386 49 63 204 386 49 63

Archytas used the natural major third, ratio 4:5 = 386 cents, and then added two different quartertones of 49 cents (35:36) and 63 cents (27:28). His preference is clearly for superparticular ratios, but his time did not realize as yet that ratios as large as these could have no musical value. Here Pythagorean numerical mysticism rules alone.

EXAMPLE NO. 13.

The Enharmonic of Aristoxenos. (ca. 330 B.C.)
Intervals: (Barbour, page 16, table 2)
Tones: e"——c"——c"-¼——b'——a'——f'——f'-¼——e'
Cents: 408 45 45 204 408 45 45

Aristoxenos works with the Pythagorean major third, ratio 64:81 = 408 cents; then he splits the remaining Pythagorean semitone of 90 cents into two equal quartertones. His scale is in complete agreement with the orthodox example shown above.

EXAMPLE NO. 14.

The Enharmonic of Eratosthenes. (ca. 230 B.C.)
Intervals: (Barbour, page 16, table 3)
Tones: e"——c"——c"-¼——b'——a'——f'——f'-¼——e'
Cents: 386 74 38 204 386 74 38

This scale uses the natural major third 4:5, then subdivides the remaining wide semitone of 112 cents into two strikingly contrasting intervals; the first is twice the size of the second one. It is a very sophisticated tetrachord, in spite of the use of superparticular ratios (23:24 = 74 cents; 45:46 = 38 cents.) What could have been Eratosthenes' guiding principle: the preoccupation with superparticular ratios, or the desire to create two sub-semitonic intervals in the ratio 1:2? One thing seems to be certain: at the time of Eratosthenes Greek music theory did not care for uniform intervals. The more variety there was within the tetrachord the better.

EXAMPLE NO. 15.

The Enharmonic of Didymos. (ca. 30 B.C.)
Intervals: (not in Barbour)
Tones: e"——c"——c"-¼——b'——a'——f'——f'-¼——e'
Cents: 386 56 56 204 386 56 56

A fully orthodox Enharmonic with the natural major third 4:5 and two equal quartertones. The historical development would seem to indicate that in the first century B.C. Eratosthenes' sophistications had been abandoned, either because musical practice did not care for them, or because they never achieved much more than theoretical stature.

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Fourth Group: DIATONIC TETRACHORDS

The Diatonic Genus has already been discussed above. Its theoretical structure in orthodox intonation would be:

	Tone	plus	Tone	plus	Semitone	=	Perfect Fourth
Cents:	204	plus	204	plus	90	=	498

EXAMPLE NO. 16.

The Diatonic of Eratosthenes (ca. 230 B.C.)
The Diatonic Ditoniaion of Ptolemy (ca. 140 A.D.)

These two intonations are identical. The intervals are listed in Barbour, page 19, table 15, and page 20, table 19.

Intervals:
Tones: e"——d"——c"——b'——a'——g'——f'——e'
Cents: 204 204 90 204 204 204 90

This is the orthodox diatonic scale in strict Pythagorean intonation, with the major whole tone 8:9 = 204 cents, and the minor semitone of 90 cents. No other superparticular ratios are used which seems to show that as late as in Ptolemy's century this orthodox scale had fully retained its importance in practical music. In fact, it is the scale which was taken over by Medieval music theory.

EXAMPLE NO. 17.

The Diatonic of Archytas (ca. 380 B.C.)
The Diatonic Toniaion of Ptolemy. (ca. 140 A.D.)

These two intonations are identical. The intervals are listed in Barbour, page 19, table 12, and page 20, table 18.

Tones: e"——d"——c"——b'——a'——g'——f'——e'
Cents: 204 231 63 204 204 231 63

Here the stress is on the wide major second, ratio 7:8 = 231 cents which narrows the "semitone" down to 63 cents (27:28). The urge to create a variety of major seconds while adhering to superparticular ratios is unmistakable. Apparently it did not matter that the diatonic principle with the traditional semitone was all but sacrificed to an interval close to a quartertone. Archytas uses the 63 cents interval in **all** his scales—a fact that invites speculation either on Archytas' preferences or on the popularity of the interval in Greece around 380 B.C.

EXAMPLE NO. 18.

The Diatonic Hemiolon of Ptolemy. (ca. 140 A.D.)
Intervals: (Barbour, page 21, table 21).

Tones:	e"——d"——c"——b'——a'——g'——f'——e'
Cents:	182 165 151 204 182 165 151

The Hemiolon (Greek, = one and one half, i. e. in this case 1½ semitones) is one of the sophisticated chroai in Greek scale building. The introduction of two ¾ tones of similar size gives this scale a particularly "exotic" quality which was certainly inspired by Asian sources. The diatonic character is nominally retained, because for all practical purposes the three tetrachord intervals are all close to minor whole tones in size. As can be expected with Ptolemy, his mathematical genius succeeded in constructing this scale with three small and immediately neighboring superparticular ratios to make up the required 498 cents of the traditional tetrachord.

8:9 = 204; 9:10 = 182; 10:11 = 165; 11:12 = 151.
182 plus 165 plus 151 = 498 cents.

This is a triumph of mathematical theory in Ptolemy's time! It is impossible, however, to make any guesses as to the musical value and practical use of this scale.

EXAMPLE NO. 19.

The Diatonic Malakon of Aristoxenos. (ca. 330 B.C.)
Intervals: (Barbour, page 19, table 13).

Tones:	e"——d"——c"——b'——a'——g'——f'——e'
Cents:	267 142 89 204 267 142 89

The chroma malakon is one of the chroai of the regular scale genera which testify to the Oriental links in Greek music. The word "malakos" means "soft". The idea, apparently, is to "soften up" the rigid diatonic intonation with its five equal major seconds. By calling the above tetrachord diatonic, Aristoxenos makes it quite clear that he considers the intervals of 267 and 142 cents both as major seconds. The ratio is 6:7 for the larger interval (267 cents) and 19:20 for the semitone (89 cents). The ¾ tone has no superparticular ratio.

It seems hopeless to speculate upon what the "softening" process actually wanted to achieve in practical performance. The scanty information available from the figures points out but one fact: the chroma malakon had to have a major second considerably larger than 204 cents, and another major second considerably smaller than 204. The only possible interpretation of this fragmentary evidence is that musicians in Aristoxenos' time must have thought of the diatonic 204 cents interval as harsh or hard, and that it could stand some softening. This points again towards Asia with its variety of "softer" seconds. It also suggests that as late as in Aristoxenos' lifetime (ca. 330 B.C.) the interval of 204 cents—an orthodox Pythagorean tuning, and a superparticular ratio (8:9) —may have sounded rigid and possibly "foreign" or artificial to Greek ears. Such an assumption is, of course, risky because it implies that Greek musical tastes and practices were strongly influenced by Asian sources and tastes 200 years after Pythagoras, even 100 years after Pericles. But the above acoustical evidence appears to support this conjecture.

EXAMPLE NO. 20.

The Diatonic Syntonon of Aristoxenos. (ca. 330 B.C.)
Intervals: (Barbour, page 19, table 14.)

Tones:	e"——d"——c"——b'——a'——g'——f'——e'
Cents:	217 192 89 204 217 192 89

This last example of Greek scale and tetrachord intonations demonstrates the continual experimentation of the theorists with the Pythagorean major third (408 cents). The objective appears always to be the splitting of the third into two **unequal** whole tones which would be contrasting not only among themselves but also with the disjunct whole tone between the two tetrachords. At this comparatively early period in the history of Greek music theory, superparticular ratios play no significant role as yet. Accordingly, Aristoxenos' syntonic seconds are based on ratios such as 15:17 (217 cents) and 17:19 (192 cents). The result of the tuning—as no doubt it was meant to be—is a strong and characteristic tension between the various whole tone steps. Whenever the Pythagorean whole

tone at 204 cents could be avoided or substituted by a contrasting interval, the theorists did so. This fact could be considered as further supporting evidence for the conclusion arrived at in Example no. 19 above.

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Most people studying Greek music theory for the first time—be it in its original version or in the misunderstood and distorted versions of Medieval and Renaissance theory—cannot help wondering about the enormous importance the **modal** structure had within the framework of a given key or scale. To our modern reasoning and ear these modes are almost meaningless; what great difference does it make whether, e. g. in the key of C-major, a melody is conceived in the range e' - e (Dorian) or d' - d (Phrygian)? The only difference, obviously, is the position of the two semitones within the scale, and besides that, possibly, what scholars consider the center of melodic gravity within the range of a given piece, or rather within the range of a given mode that scholars try to apply to that piece in their attempt at modal interpretation. This center of gravity—called mese by the Greeks—plays an important part in all modern thought on Greek theory, and in all attempts to interpret the structure of the musical relics. In fact, the fundamental characteristics of any mode as defined by most authors and textbooks are the range of the scale, its mese, its ending tone (finalis).

To his never-ending surprise this writer has never come across a really convincing and satisfactory explanation by any author for the tremendous importance of modes and modal structure in Greek music theory and practice. Range, mese, and finalis are certainly not plausible enough to explain a system as complex and subtle, as highly developed and diversified as Greek modal doctrine. There is reason for wonderment because a convincing explanation is simple.

Most writers seem to think inadvertently of Greek modes in terms of simple diatonic scales; many of them seem even to think, unconsciously, of such scales in terms of equal temperament. Under such conditions modes and modal structures become indeed insignificant: location of the semitone in equal temperament, as the only criterion, makes a diatonic mode an empty form without meaning. But even in Pythagorean diatonic intonation, with its only two intervals of 204 and 90 cents, a mode has not much characteristic color of its own to distinguish it markedly from other modes. It is for this reason that Medieval and Renaissance music failed to develop a modal theory that could really be called significant, original in its own right and independent from Greek thought. For the same reason modal writing began to decrease with the introduction of effective temperaments into the musical practice of the Renaissance period, and it disappeared altogether with the general acceptance of equal temperament in the Baroque era.

These are simple and clear facts we would like to see in print in some of the textbooks and musical histories in use in our colleges and conservatories.

As soon as we realize, however, that the ordinary Pythagorean Diatonic was only one of possibly dozens of Greek intonations and that it was probably not used very frequently or at least not predominantly, a mode takes on an overwhelming significance. Let us take, for instance, the Diatonic of Archytas:

Cents: 204 231 63 204 204 231 63

or the Diatonic Syntonon of Aristoxenos:

Cents: 217 192 89 204 217 192 89

Here we have three or even four completely different intervals within the scale, or even within the tetrachord, two or three different sizes of whole tones and innumerable varieties of composite thirds, fifths, sixths, sevenths. Whether we start and end such a scale on the first, third, or fourth tone will make an enormous difference: the character of the scale and of the melody conceived in it becomes so vastly changed from mode to mode that one is almost dealing with an altogether different scale.

If we proceed from these comparatively unsophisticated variations of the diatonic principle to the chroai of the diatonic genus or to the chromatic and enharmonic genera, every change of mode within the scale creates not only a new "scale", but it seems almost to change the national, racial or territorial character of the scale and melody.

These facts—hardly ever stated—make it very probable that mode and scale were identical in early Greek theory; each different mode was a different scale in the beginning—an idea which has already been formulated by Curt Sachs for different reasons.

The development may have gone along these lines: a particular intonation was taken over from one of the West Asian tribes and named after them: Lydian, Phrygian, etc. Later on experimentation started, and this intonation was "superimposed" on an already absorbed or "domesticated" scale, adding a new "mode" to its previously known and used scale principle. Gradually the material and its available variants became so vast in numbers and selective potential that confusion arose. In one single diatonic intonation there were 12 possible keys and seven possible modes each, amounting to 84 varieties. Add to this a few dozen other diatonic, chromatic and enharmonic tunings with their various chroai, and the number of possible commutations might have come well over 10,000 scales, keys, modes and shades. Such must have been the state of affairs when, in the middle of the fourth century B.C. Euclid began to bring order into the chaos by selecting, eliminating and forming. It should be noted, however, that his "perfect system" concerned itself only with modal and scale structure; he left untouched the freedom of intonation or tuning in its varieties. Had he done so successfully, he might have destroyed the fundament of modal variation—a development that did not take place until much later in Western Europe.

* * * * * *

It seems appropriate to deal here with another erroneous notion implicit in many textbooks and fixed in the minds of many musicians: the idea that the Pythagorean comma bothered Greek music theory and practice greatly through hundreds of years, and that the Greeks did not know "how to get rid of it".

If we assume, with a reasonable degree of justification, that the 12-step Pythagorean cycle was not completed much earlier than 450 B.C., the nature of the comma could not have been known to mathematicians and felt by musicians before that date. But as early as 380 B.C. Archytas constructed his first tetrachords; quite possibly tetrachords were known even before Archytas, but we have no documentary evidence so far of earlier constructions. Archytas' scales are already built in the tradition which remained unchanged through most of Greek music theory:

	Perfect Fourth	plus	Major Second	plus	Perfect Fourth	=	Octave
Cents:	498	plus	204	plus	498	=	1200

All intervallic subdivisions and changes were henceforth calculated by shiftings within the perfect fourth of the tetrachord. The skeleton of the fourth, of the perfect fifth (498 plus 204 = 702 cents) and the octave remained untouched. It follows that at least since Archytas the comma had been "gotten rid of", both in mathematical theory and musical practice. In fact it is very doubtful whether it ever presented a problem to the performing musician. If it did, it could have been only for about 70 years, between 450 and 380 B.C.

The crux of the matter is that enharmonic changes are the main, or even the only cause of comma troubles, and they will occur only in keys with more than three accidentals, as demonstrated in examples no. 2 and 3.

Thus, the comma became again, or possibly for the first time, a real problem in early Renaissance music in Western Europe. In this area and period the simple diatonic scale in Pythagorean tuning remained, for all practical purposes, the only bit of the Greek heritage of intonations Western Europe was able and willing to absorb. This is a strange fact if we consider all the preoccupation with Greek music theory that is so characteristic of Medieval and Renaissance thought on music.

We have good reason to be happy about this ignorant and stupid self-restriction which forced the West to develop polyphony as the only remedy against the monotonous boredom of diatonic monophony. Had Western Europe taken over all or most of the intonations and scales which made up the expressive riches of ancient Greece, we might to this day not know the harmonies, the polyphonous wonders and many other achievements of our musical culture.

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SIDE 2

Fifth Group: GREEK RELICS

EXAMPLE NO. 21.
The Skolion of Seikilos.

Intonation in Ptolemy's Diatonic Hemiolon (cf. Example no. 18).

After Davison-Apel, p. 10 Transposed one tone down from Davison-Apel's transcription.

The choice of intonation in Ptolemy's Diatonic Hemiolon is not quite as arbitrary as it might seem. This charming little drinking song is generally attributed to the first or second century A.D., and its character is clearly diatonic. Thus one of the diatonic scales of Ptolemy who flourished around 140 A.D. seems appropriate. To give at the same time an impression of Greek chroai, the chroma hemiolon was selected. There is, of course, no certainty whatever that the Skolion was actually ever sung or played in this intonation, but a possibility does exist.

There is one great difficulty that handicaps all attempts to make plausible guesses or conjectures about performance practice in ancient Greece and its colonial territories: while a substantial amount of scholarly literature and theoretical treatises has been preserved and come down to our time, the sources of actual music preserved in notation are extremely scanty. There are not more than eleven relics in all; none of them are very long, and most of them are fragmentary.

To reconcile the enormous complexity of Greek musical theory against these few sources of actual music is impossible. The available material is much too limited to cover even a small amount of the frequently obscure rules and reasonings of theoretical literature.

As a consequence, opinions, interpretations and transcriptions of scholars specializing in Greek music differ widely. More often than not they are based on ingenious conjectures, deductions and hypotheses rather than on facts or evidence both of which are missing.

As will be seen in some of the next examples, the transcriptions and interpretations even of two leading authors in US—Curt Sachs, 1943; and Davison-Apel, 1946;—are far from agreed on various fundamental points of argument. The present commentary to a recording is no place to take sides or to enter into discussion on the merits of any point of view on the modal and other structural aspects of the recorded pieces. The aim of this recording is to present living sound, in the hope that this might contribute to a better understanding of Greek music theory the same way as the learned arguments of scholarly philology or musicology.

It was for purely practical reasons that the transcriptions by Davison-Apel were chosen for this recording: they are printed in their entirety in the named version, and they are available in most libraries, colleges and conservatories in this country.

EXAMPLE NO. 22
The Skolion of Seikilos.

After Davison-Apel. Repeated in Ptolemy's Diatonic Toniaion, or in the Diatonic of Archytas (cf. Example no. 17).

This variant of intonation is selected to show the effects of major whole tones of different sizes. It may be assumed that this intonation must have retained a certain popularity over a considerable period of time; Archytas computed it around 380 B.C. and Ptolemy reports it again in his time, around 140 A.D.

The transposition one tone down from Davison-Apel's transcription has no other reason than to retain the Dorian octave (cf. col. 5) in C-major throughout all examples of the recording—a device that simplifies listening and recording and avoids the involvement with questions of modal structure.

Davison-Apel interpret the Seikilos Skolion as composed in the Phrygian octave species d' - d, "transposed a tone upwards". Sachs, in his analysis, says that "the melody is distinctly Phrygian . . . in the range e' - e."

EXAMPLE NO. 23.

Hymn to Helios.

ca. 130 A.D. After Davison-Apel, p. 9. Intonation in the Diatonic Malakon of Aristoxenos. (cf. Example no. 19)

Sachs interprets this Hymn to the Sun God as written in the Mixolydian key and mode, range f' - f, with one flat. Davison-Apel give no interpretation. Both authors transcribe in F-Major, but the respective transcriptions show differences in rhythmic interpretation and, consequently, in time values. Sachs gives the following tetrachord for his transcription which has been used for the recorded intonation:

a——g——f——e // d——c——B♭——A

The Oriental character of the piece in this intonation is unmistakable; it is attributed to Mesomedes of Crete who lived around 130 A.D.. As the melody is basically diatonic, a diatonic tuning had to be selected.

EXAMPLE NO. 24.

repeats the Hymn to Helios in strict diatonic intonation after Eratosthenes and Ptolemy in order to permit a comparison without the admixtures of the chroma malakon. Here all whole tones measure 204 cents, the semitones 90 cents each.

Even in this tuning which comes close to modern equal temperament, the "non-European" characteristics are evident. While the ear is no more preoccupied with digesting unusual intervals, we become aware of certain rhythmical peculiarities and of certain interval steps which do not normally occur in European music prior to about 1900. The composition and the period seem to be near a borderline across which Asian features are amalgamating with new and distinctly Western idioms which must have begun to form then or somewhat earlier. Especially interesting is the repeated "thematic" appearance of certain melismatic and rhythmical units which might be taken to indicate such early "Western" principles of form, or expression.

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Sixth Group: BASIC GREEK INTERVALS.

The following examples demonstrate some of the more important intervals of Greek theory. In their selection preference was given to those intervals which had significant influence on the development of later Western theory, and to those which are most conspicuously dealt with in the traditional textbooks, such as the commas of Pythagoras and Didymos. The most important interval, no doubt, is the major third whose intonation is still subject to lively arguments even in contemporary music theory and practice.

EXAMPLE NO. 25.

The Major Third and Triad in Equal Temperament.

Tones:	C		E		G
Pitches:	0		400		700
Intervals:		400		300	

This is the intonation we are used to in all keyboard instruments, the harp, glockenspiel etc.,—i. e. in all instruments with fixed tuning for every tone throughout every performance. We are fully accustomed to these tempered pitches and find them satisfactory without consciously listening to them.

EXAMPLE NO. 26.

The Major Third and Triad in Just Intonation.

Ratio:	4	:	5	:	6
Tones:	C		E		G
Pitches:	0		386		702
Intervals:		386		316	

According to many theorists this is, or ought to be, the best or even ideal intonation. With admirable steadfastness and faith, tone physiology and musical psychology keep repeating that the small superparticular ratios, such as the above 4:5 and 5:6 for the major and minor thirds, are the most satisfactory ones from the physiological and psychological point of view, because they are replicas of the natural series of partial tones. (400:500:600 cps, for example, = major triad.) The idea at the bottom of this "axiom" is obviously that inherent, innate in the human ear, nervous system and psychic constitution, is a sort of physical response tuned to certain wave lengths or vibration ratios which causes us to feel most satisfied, physically and esthetically, with intonations that are based upon the natural harmonic series; in other words, our hearing sense is in full agreement with certain acoustical laws.

The following example will easily explode this theory. Most musical people, on concentrated listening to this narrow natural third of 386 cents, are dissatisfied and find the pitch offensively flat. In comparison, they feel the minor third of 316 cents in this triad is much too wide, the difference being only 70 cents for a semitone. In many pre-release hearings of this recording for test purposes, we found this view confirmed by about 90% of roughly 120 listeners, all of them professional musicians or advanced music students.

The consequence of this little experiment in musical psychology, and the reason for this general reaction is simply this:

Those intonations are the most satisfactory ones to our ears to which we are normally exposed and accustomed. There are no **physiological** laws behind our reactions to certain intervals, and our response is entirely conditioned by psychological facts, or in plain language: by habit. This insight should also be helpful to overcome our intolerance and prejudices when we are confronted with certain "exotic" scales and intervals which sound strange or even offensive to our ear. "Exotic" intervals are not "off pitch". Other races and nations got used, throughout their musical histories, to pitches and intonations different from ours. Any serious student of non-Western music experiences a significant change of attitude in this respect. What sounded at first unbearable and tormenting, becomes finally quite acceptable or even attractive. This process of converting listening habits, however, takes normally several years of acclimatization to unusual intervals.

EXAMPLE NO. 27.

The Major Third and Triad in Pythagorean Intonation.

Tones:	C		E		G
Pitches:	0		408		702
Intervals:		408		294	

Pre-release hearing tests with musicians and music students showed that the very wide third of 408 cents sounded better to them than the just intonation of 386 cents. Some of the listeners even preferred the Pythagorean third to the 400 cents of equal temperament. In the full triad, however, they disliked the narrow minor third of 294 cents.

There is much confusion among concert-goers about the actual intonation in modern symphonic performances. Many people seem to believe that equal temperament governs throughout, but that is true, to a certain degree, only when there is a piano or the tempered harp in the score. In the absence of such instruments intonation shifts from moment to moment, and major thirds will most frequently sound at wide pitches, 400 cents or even above. Newcomers to the brass section of a symphonic body have a tendency to intone narrow major thirds because the acoustical properties of brass instruments are based on the natural harmonic series. Usually it takes some time for them to get used to the wide thirds of the string players: the tuning of the violins, violas and cellos in fifths tends to make the string section intone wide major thirds close to Pythagorean pitch.

We all know the terrible clash of intonations occurring when a concert hall organ joins fortissimo with the sounds of a symphony orchestra. This shows how difficult it is for a symphonic body to adhere to equal temperament. Piano and harp, being less powerful in volume, do not stress differences of intonation as much as a full organ playing at great volume. But a keen listener will notice that in powerful chordal passages as they often occur at the end of the finale movement in piano concertos, certain pitch differences between orchestra and soloist become evident.

EXAMPLE NO. 28.

Minor Third and Triad in Equal Temperament.

Tones: C——E♭——G
Pitches: 0 300 700
Intervals: 300 400

This is the tuning we are used to hearing in our time on keyboard instruments and harps.

EXAMPLE NO. 29.

Minor Third and Triad in Just Intonation.

Tones: C——E♭——G
Pitches: 0 316 702
Intervals: 316 386
Ratios: 5:6 4:5

In a limited number of pre-release tests—when the full triad was sounded—most listeners became aware **first** of the very narrow major third E♭——G (386 cents), and only then did they notice how wide this minor third is (316 cents.) The difference between the major and minor interval, 70 cents, is indeed very small as compared with the 100 cents of equal temperament. Could this reaction of most listeners be taken as supporting evidence for our assumption that our contemporary ears dislike the narrow major third of just intonation?

The comparison at the end of the example is between the minor thirds in just intonation and equal temperament:

{ C——0——————
{ E♭——316——————300

EXAMPLE NO. 30.

Minor Third and Triad in Pythagorean Intonation.

Tones: C——E♭——G
Pitches: 0 294 702
Intervals: 294 408
Ratios: 16:27 64:81

Here the impression is similar as in Example no. 29. When the triad is sounded, one seems to notice **first** the wide major third (408 cents), and only as a consequence of this tone sensation the narrowness of the minor third (294 cents) is realized. This might be interpreted to mean that, unconsciously, we are inclined to judge the quality of any triad, major or minor, by the size of the major interval.

The comparison at the end of the example is between the minor thirds in Pythagorean intonation and equal temperament:

{ C——0——————
{ E♭——294——————300

EXAMPLE NO. 31.

The Pythagorean Comma.

In unison (after seven octave transpositions, see Table no. 1, step 12)

Tones: C——B♯
Pitches: 0 24

In the octave (after six octave transpositions, see Table no. 1, step 12a)

Tones: C——B♯'
Pitches: 0 1224

It should be noted that many persons have at first some difficulty in determining whether the higher B♯' is flat or sharp; it is, of course, 24 cents sharp. This "acoustical illusion" is quite common, before one gets used to micro-intervallic differences. The comparison here is between 1224 and 1200 cents. It is amusing to observe how low the perfect octave seems to sound in this context.

EXAMPLE NO. 32

Diatonic Seconds (Pythagorean Intonation).

Tones: C——D——E
Pitches: 0 204 408
Intervals: 204 204

Here the major third in Pythagorean intonation (64:81) consists of two equal diatonic seconds, ratio 8:9, also a Pythagorean interval.

EXAMPLE NO. 33.

Syntonic Seconds.

Tones: C——D——E
Ratios: 8 : 9 : 10
Pitches: 0 204 386
Intervals: 204 182

The major third in natural intonation calls for two different seconds which are called major (204 cents) and minor (182 cents) whole tones. The discovery of these narrow "syntonic" seconds is usually ascribed to Didymos, but as we have seen above (col. 3), the natural third of 386 cents, and consequently the "syntonic" second of 182 cents (being the difference between the third of 386 and the diatonic interval of 204), were already known to Archytas, with the superparticular ratios for the two different seconds of 8:9 and 9:10.

EXAMPLE NO. 34.

The Syntonic Comma.

This comma is the difference between the diatonic and the syntonic seconds: 204 — 182 = 22 cents. It appears, of course, also as the difference between the Pythagorean and the natural major third: 408 — 386 = 22 cents. The example gives the tones in the following order:

(a) 0——204——386; (b) 0——204——408;
(c) 0——204——386; (d) 0——204——408.
(e) 0——386——408; (f) 0——386——408.
(g) { 0—————— (h) { 0——————
{ 386——————408 { 386——————408

EXAMPLE NO. 35.

The wide Second, 7:8 = 231 cents.

This interval played a certain role in Greek theory because it has a superparticular ratio; it also supported the strong inclination of Greek musicians and theorists to build their tetrachords of seconds as different as possible. (Cf. Ptolemy's tetrachord 9:10, 10:11, 11:12 = 182 plus 165 plus 151 = 498 cents).

In Chinese music theory this second has a particular importance: a zither-type string instrument, called ch'in, uses this interval of 231 cents at the first stopping stud on each of its seven strings, and there is some evidence that the famous "short" fifths of Chinese music were 693 cents wide and composed of three equal intervals 7:8 = 231 plus 231 plus 231 = 693 cents. This is neither a tetrachord (which spans the interval of a perfect fourth) nor a pentachord (which spans a perfect fifth but has five tones within this compass instead of these four.) It is a typically Chinese concept of ancient times and may have influenced the later Indonesian scale of slendro (= wide intervals) which consists of five equal intervals of 240 cents each within the octave (1200 cents). The example demonstrates this theoretical Chinese construction:

Tones: C D+ E++ G
Pitches: 0 231 462 693
Intervals: 231 231 231

In hearing this example, practically all listeners, not only beginners, will become victims of another acoustical illusion. Although we have three precisely equal intervals, we feel we are hearing about this sequence:

C——D+——E++——————G
Wide Second Wide Second Narrow Minor Third

This illusion is due to the fact that we are used to scales subdividing the compass of a fifth into four interval steps instead of three. After having heard two wide major seconds there follows the fifth, and unconsciously we conclude that the last step must have been a (narrow) minor third, not a diatonic step, because one interval step is missing. It takes very concentrated listening, almost an act of will power, to overcome this illusion. Here is another illustrative instance to show how much our aural reactions and impressions are conditioned by habit.

Seventh Group: GREEK MUSICAL RELICS.

EXAMPLE NO. 36.

First Delphic Hymn, Section A.

Transcription after Davison-Apel, page 9. Intonation in the Diatonic of Eratosthenes (tonus = 204 cents, semitonus = 90 cents).
Here is one of the cases where transcriptions and interpretations vary substantially between the versions of various scholars.

	Sachs	Davison-Apel
Transcribed key:	E♭-Major	D-Major
Transcribed accidentals:	3 flats	2 sharps
Range:	a♭'——e♭	f♯'——f♯
Mode:	Phrygian	Dorian, transcribed a whole tone upward
Key:	Phrygian	- - -
Mese:	c'	b

Sachs gives a tetrachord for his transcription of Section A which, however, keeps a second separate tuning "in reserve".

Fig. 5

This interpretation means that in various parts of the piece one would have to shift from the conjunct tetrachord of unit (2) to the disjunct tetrachord of unit (3) and vice versa. Sachs mentions that in later Greek theory the principle of symmetrical tetrachord construction was abandoned, and that asymmetrical units make their appearance. This development means, in other words, that the rigid tetrachord system was liberalized into a free scale with occasional chromatic alterations.

In order to make one of our symmetrical tetrachords applicable to the composition on hand, a tetrachord was designed which claims no theoretical or analytical value; its sole purpose is to serve as a tuning basis for this recording, its only merit is that it fits the tones in the recorded composition. The First Delphic Hymn is usually dated around 138 B.C. That is rather late in the history of Greek theory and may well mean that in practice the symmetrical tetrachord structure had already been broken up.

Fig. 6

In this construction the principle of alternate disjunction and conjunction is maintained. The C-natural between units (1) and (2), then, is to be interpreted as a chromatic alteration of C♯, serving occasionally as an alternative. Sachs, in his transcription, speaks of a modulation into another key, lower by one semitone. Thus, in principle, the two tetrachords are equivalent; they differ, however, in form and practical application.

The melodic structure of Section A is fairly close to what we are inclined to consider as "Western" music. The 5/8 rhythm, however, is doubtless of Asian origin. In Greek musical practice it represents the so-called Cretic meter (cf. "Chronos" in Apel's Harvard Dictionary of Music) which was very common in Greek poetry.

EXAMPLE NO. 37.

First Delphic Hymn, Section B.

Transcription after Davison-Apel, page 9. Tuning in the Chromatic Tonikon of Aristoxenos.

Tetrachord: 316 plus 93 plus 89 = 498 cents.

This tuning is a fairly orthodox chromatic intonation. It is well suited for the strongly chromatic character of Section B which is in striking contrast to the first Section because of the extensive use of semitonal progressions.

There are again fundamental differences in the interpretations by our two scholarly sources:

	Sachs	Davison-Apel
Transcribed key:	A♭-Major	D-Major
Transcribed accidentals:	4 flats	2 sharps
Range:	A♭'——A♭	- - -
Mese:	f'	- - -
Key and mode:	Hypermixolydian with modulation into conjunct parallel	- - -
Genus:	Enharmonic	Chromatic

Sachs gives two different tetrachords for this Section in order to meet the varying needs of the score which are continuously changing. We are inclined to believe that this piece no longer follows any tetrachordal system, and that any attempt to construct **one** model of tetrachord for the whole section must fail. Two different types of tetrachords, however, used alternately for the same piece mean, of course, repeated re-tuning during performance.

Again for purely practical purposes we designed something resembling a tetrachord in order to make the piece playable in any of our tetrachord intonations without re-tuning:

Fig. 7

Here we have three symmetrical units of a chromatic tetrachord, with the sequence: minor third—semitone—semitone. Also, as is prescribed by theory, the units follow each other as alternating conjunction and disjunction. The trouble with this construction is that at the point of disjunction there is a semitone instead of a whole tone (from b to a♯). Yet, for practical application of a chromatic tetrachord it will do, and it fits the conditions of the composition. If anyone can devise a better tetrachord for Section B that is playable without re-tuning during performance, we shall be glad to accept it.

The differences in mood, expressive content, and melodic structure are so striking it is hard to believe that the two Sections of the First Delphic Hymn were created at the same location, in the same period, and possibly even by the same composer. As there is very little known about the original source, speculation about the nature and causes of these differences seems to be futile. The only permissible conclusion appears to be that Greek composition in the second century B.C. must have covered a wide range of technical and expressive means.

EXAMPLE NO. 38.

First Delphic Hymn, Section B.

Repeated in the Chromatic Malakon of Aristoxenos.

As there is no certainty that Section B was always performed in a simple chromatic tuning, we give here, as a last example, the same Section in one of the chromatic shades, the chroma malakon.

This Hymn is the most important and substantial relic of Greek musical culture, and it should get the benefit of another variety of chromatic intonation. One of the objectives of all these demonstrations was to reveal and to stress the deep indebtedness of Greek music to West Asian sources. The selection of a chromatic malakon for this piece is by no means a trick to achieve this objective. The Greek chroai were an important part of their theory and musical practice, and they were widely used in performance. If the chroma malakon stresses the Asian fundaments more than some other intonations, it is more impressive and helpful in the attempt to drive this point home.

No highly developed musical civilization, mainly based on melodic and rhythmical principles, was ever able to flourish without tremendous complexities and sophistications in these two media. Western music was content with the diatonic and chromatic scale in one single intonation and had to develop other complexities and sophistications: harmony, polyphony, orchestration. Certain modern composers try to combine these two different worlds and to introduce Asian melodic and intonation complexities into the patterns of modern Western music, which are intricate enough as it is. The chances are that this will be more than Western ears and minds are able to absorb. In the arts one can rarely have everything at once without becoming unintelligible.

Engineer's Report:

For best results phono equalization should be set for NAB curve (turnover at 500 cps, roll-off at 10,000 cps 16 db). Playback recommended at moderate volume levels whenever possible. These disks are pressed of 100% pure vinylite which reduces surface noise almost to the vanishing point.

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